## BE 150 Spring 2018 Homework #2 Due at the start of lecture, April 18, 2018.

Problem 2.1 (Design principles for toggles, 30 pts).

Consider two components, A and B, which regulate each other. A may activate or repress B, and B may activate or repress A. There are three architectures, two with positive feedback and one with negative feedback, shown in Fig. 1.



Figure 1: Three toggle-like architectures.

- a) A circuit can behave like a toggle if it has two stable steady states, one with A high and B low and another with B high and A low. Only one of the architectures in Figure 1 can function as a toggle. Which one? Explain in words and sketches why *only* the one you chose can be a toggle.
- b) A and/or B may show ultrasensitivity regulation, which we describe with a Hill coefficient, *n*, greater than one. Show that without ultrasensitive regulation, even the architecture you chose cannot have toggle behavior.

Problem 2.2 (Comparison of incoherent feedforward loops, 30 pts).

In Fig. 2, we show two types of incoherent feedforward loops, type 1 (I1-FFL) and type 4 (I4-FFL). Interestingly, the I1-FFL is found much more often in naturally occurring gene regulation networks than the I4-FFL. Some speculate that this is because expression level of Z is sensitive both to changes in X and Y in the I1-FFL, while the level of Z is sensitive only to changes in X in the I4-FFL (thus making Y dispensable). In this problem, you will investigate this claim.



Figure 2: Two of the four types of incoherent feedforward loops.

a) Assume the I1-FFL circuit is at some nonzero steady state. Suddenly, the amount of X goes to zero. Sketch the response of Z, assuming AND logic for

the regulation of Z expression. Do this again again assuming OR logic. Comment on any features of the plot you think are noteworthy. *Hint*: AND logic in the case where X is an activator and Y is a repressor means "X AND (NOT Y)" and OR logic means "X OR (NOT Y)."

- b) Repeat (a) except for the I4-FFL circuit.
- c) Repeat (a) and (b) except with the amount of Y suddenly going to zero.
- d) Are the discrepancies between the I1-FFL and the I4-FFL more pronounced when they have AND logic or OR logic? Explain.

## Problem 2.3 (Autoregulation in a C1-FFL, 40 points).

This problem is based off of problem 4.3 from Alon's book. The type 1 coherent feedforward loop (C1-FFL) is another motif that is found in large numbers in naturally occurring networks. In Fig. 3, we show two C1-FFL networks in which the regulator Y is autoregulated. In the C1-FFL with AND logic, Y shows autorepression, and in the C1-FFL with OR logic, it shows autoactivation. These "decorations" on C1-FFLs often occur in nature.



Figure 3: Decorated C1-FFLs. (a) Y is autorepressed and Z is dictated by AND logic. (b) Y is autoactivated and Z is dictated by OR logic.

- a) As we learned in lecture, the C1-FFL with AND logic shows sign-sensitive delay. Specifically, if X is suddenly turned on, the response of Z is delayed, but if X is suddenly turned off, the response of Z is instantaneous. Analyze the circuit in Fig. 3a, and compare its dynamics to the canonical C1-FFL circuit with AND logic. Specifically address how the delay time changes. Assume that the regulation of Y follows AND logic.
- b) The C1-FFL with OR logic also shows sign-sensitive delay. With OR logic, though, there is no delay when X is suddenly turned on, but rather delay when X is suddenly turned off. Analyze the circuit in Fig. 3b, and compare its dynamics to the canonical C1-FFL circuit with OR logic. Again, address how the delay time changes, this time assuming that the regulation of Y follows OR logic.